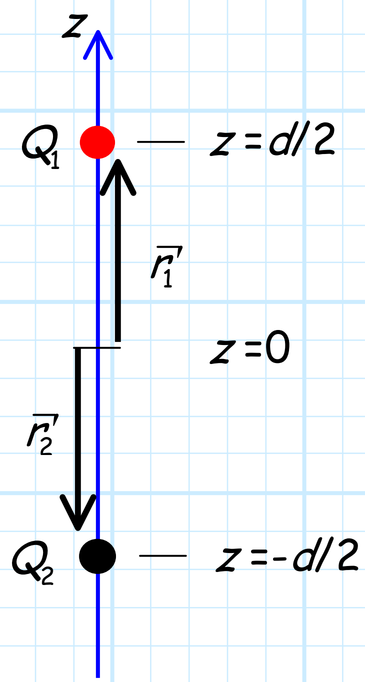
Example: The Electric Dipole

Consider **two** point charges (Q_1 and Q_2), each with **equal magnitude** but **opposite sign**, i.e.:

$$Q_1 = Q \quad \text{and} \quad Q_2 = -Q \quad \text{so} \quad Q_1 = -Q_2$$

Say these two charges are located on the z-axis, and separated by a **distance** d.



The **location** of charge Q_1= $-Q$ is therefore specified by **position vector** $\bar{r}_1' = \frac{d}{2}\hat{a}_z$

The **location** of charge Q_2= $-Q$ is therefore specified by **position vector** $\bar{r}_2' = \frac{-d}{2}\hat{a}_z$

The Electric Dipole

We call this charge configuration an **electric dipole**. Note the **total charge** in a dipole is **zero** (i.e., $Q_1 + Q_2 = Q - Q = 0$). But, since the charges are located at different positions, the electric field that is created is **not** zero !

Q: *Just what **is** the electric field created by an electric dipole?*

A: One approach is to use **Coulomb's Law**, and add the resulting electric **vector** fields from each charge together.

However, let's try a different approach. Let's find the **electric potential field** resulting from an electric dipole. We can then take the gradient to find the electric field !

Note that this should be relatively **straightforward**! We already know the electric potential resulting from a **single** point charge—the electric potential resulting from two point charges is simply the **summation** of each:

$$V(\bar{r}) = V_1(\bar{r}) + V_2(\bar{r})$$

where the electric potential $V_1(\bar{r})$, created by charge Q_1, is:

$$V_1(\bar{r}) = \frac{Q_1}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}_1\right|} = \frac{Q}{4\pi\varepsilon_0 \left|\bar{r} - \frac{d}{2}\hat{a}_z\right|}$$

and electric potential $V_2(\bar{r})$, created by charge Q_2, is:

$$V_2(\bar{r}) = \frac{Q_2}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}_2\right|} = \frac{-Q}{4\pi\varepsilon_0 \left|\bar{r} + \frac{d}{2}\hat{a}_z\right|}$$

Therefore the **total** electric potential field is:

$$\begin{aligned} V(\bar{r}) &= \frac{Q}{4\pi\varepsilon_0 \left|\bar{r} - \frac{d}{2}\hat{a}_z\right|} - \frac{Q}{4\pi\varepsilon_0 \left|\bar{r} + \frac{d}{2}\hat{a}_z\right|} \\ &= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\left|\bar{r} - \frac{d}{2}\hat{a}_z\right|} - \frac{1}{\left|\bar{r} + \frac{d}{2}\hat{a}_z\right|}\right) \end{aligned}$$

If the point denoted by $\bar{r}$ is a significant distance away from the electric dipole (i.e., $\left|\bar{r}\right| >> d$), we can use the following **approximations**:

$$\frac{1}{\left|\bar{r} - \frac{d}{2}\hat{a}_z\right|} \approx \frac{1}{\left|\bar{r}\right|} + \frac{d\cos\theta}{2\left|\bar{r}\right|} = \frac{1}{r} + \frac{d\cos\theta}{2r^2}$$

$$\frac{1}{\left|\bar{r} + \frac{d}{2}\hat{a}_z\right|} \approx \frac{1}{\left|\bar{r}\right|} - \frac{d\cos\theta}{2\left|\bar{r}\right|} = \frac{1}{r} - \frac{d\cos\theta}{2r^2}$$

where r and θ are the **spherical coordinate** variables of the point denoted by $\bar{r}$.

Therefore, we find:

$$
\begin{aligned}
V(\bar{r}) &= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\left|\bar{r}-\frac{d}{2}\hat{a}_z\right|}-\frac{1}{\left|\bar{r}+\frac{d}{2}\hat{a}_z\right|}\right) \\
&= \frac{Q}{4\pi\varepsilon_0}\left(\left(\frac{1}{r}+\frac{d\cos\theta}{2r^2}\right)-\left(\frac{1}{r}-\frac{d\cos\theta}{2r^2}\right)\right) \\
&= \frac{Q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}
\end{aligned}
$$

Note the result. The **electric potential field** produced by an **electric dipole,** when centered at the **origin** and aligned with the ***z*-axis** is:

$$
V(\bar{r}) = \frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}
$$

Q: *But the original question was, what is the* ***electric field*** *produced by an electric dipole?*

A: Easily determined! Just take the **gradient** of the electric potential function, and multiply by -1.

$$
\begin{aligned}
\mathbf{E}(\bar{\mathbf{r}}) &= -\nabla V(\bar{\mathbf{r}}) \\
&= -\nabla\left(\frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}\right) \\
&= \frac{-Qd}{4\pi\varepsilon_0}\left[\cos\theta\ \frac{d}{dr}\left(\frac{1}{r^2}\right)\hat{a}_r + \frac{1}{r^3}\frac{d(\cos\theta)}{d\theta}\hat{a}_\theta\right] \\
&= \frac{-Qd}{4\pi\varepsilon_0}\left[\left(\frac{-2\cos\theta}{r^3}\right)\hat{a}_r - \frac{\sin\theta}{r^3}\hat{a}_\theta\right]
\end{aligned}
$$

The static **electric field** produced by an **electric dipole,** when centered at the **origin** and aligned with the ***z*-axis** is:

$$
\mathbf{E}(\bar{\mathbf{r}}) = \frac{Qd}{4\pi\varepsilon_0}\frac{1}{r^3}\left[\ 2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right]
$$

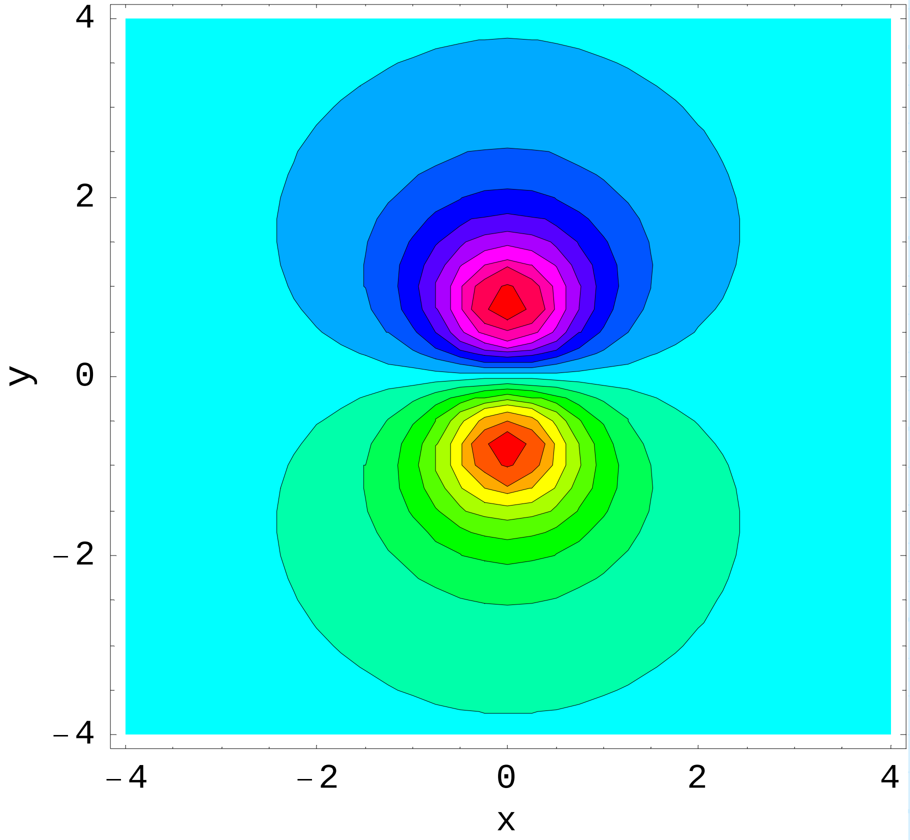
Yikes! **Contrast** this with the electric field of a **single** point charge. The electric dipole produces an electric field that:

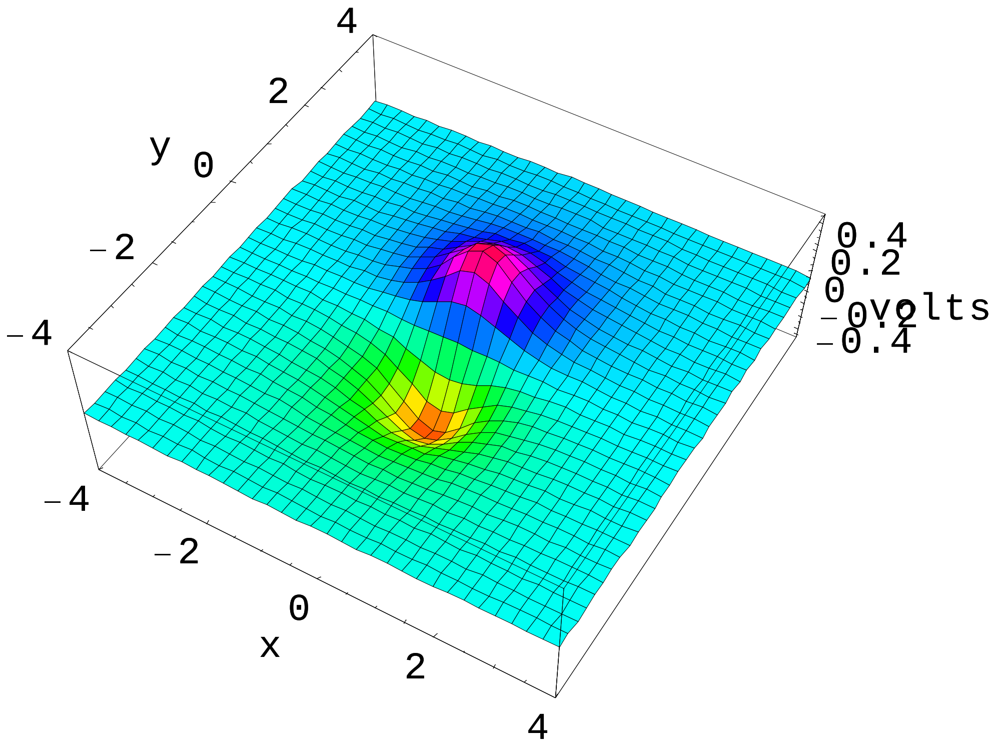
1) Is proportional to r^{-3} (as opposed to r^{-2}).

2) Has vector components in **both** the $\hat{a}_r$ and $\hat{a}_\theta$ directions (as opposed to just $\hat{a}_r$).

In other words, the electric field does **not** point away from the electric dipole!

The **electric potential** produced by an electric dipole looks like:





And the **electric field** produced by the electric dipole is: